Matthew George O. Escobido

Asian Institute of Management

123 Paseo de Roxas

Makati City 1260

Philippines

Telephone number: +632-892-4011 loc. 131

e-mail:mescobido@aim.edu

Product differentiation and the dynamics of competition

Abstract

In this paper, we are going to analyze the effects of vertical (quality) and horizontal (substitutability) differentiation in the competition of products. These two aspects of differentiation are incorporated in the utility function of the Dixit duopoly framework. We then develop a dynamic model based on Cournot competition where firms adjust their output based on the product's lifecycle. We derived a coupled equation governing the dynamics of the competition. We present the conditions for long-run coexistence of the products as well as the conditions for the success of one and the failure of the other. These conditions define a region in the quality vs. substitutability space that allows us to predict the dynamics of the competition. We investigate the resulting dynamics as the degree of differentiation between the products change.

Keywords:

Product differentiation, quality, substitutability, competition dynamics, product lifecycle

Introduction

Product differentiation is a very important product management concept that its inclusion is almost given in any product strategy. So important that the Product Development and Management Association (PDMA) made it clear that superior and differentiated new product are the number one drivers of success and product profitability (Kahn, 2005). What is not clear, however, is how do "superiority" and "differentiation" define the success of the product? Or to be more precise, how much more superior or more differentiated should a product be to succeed in the competition?

Numerous models have been made to show how differentiation affects competition between products. The usual conclusion is that differentiation reduces competition between products as it relaxes selection pressure emanating from the consumers (Bain, 1956). It thus increases the chances for the products to coexist. A usual way of illustrating this effect is by using a static Cournot competition model where product differentiation results in increase in profits in a Cournot equilibrium (Beath & Katsoulacos, 1991).

Our model builds on similar constructs with two main extensions. First is that product differentiation is introduced using Dixit's utility function (Dixit, 1979). This allows us to capture aspects of vertical differentiation (quality) and horizontal differentiation (substitutability) simultaneously. Vertical differentiation occurs when two products can be ordered according to their objective quality from the highest to the lowest. It becomes possible to say in this case that one product is "superior" to the other. Horizontal differentiation on the other hand emerges when products have different features that cannot be ordered in an objective way (Tirole, 1988). Such decomposition will help us isolate and differentiate the effects of these aspects of product differentiation.

The second extension is the consideration that competition is not static but changes over time. As such we use a dynamic version of the Cournot competition where firms adjust their respective outputs based on the phase of the product life cycle it is in. This adjustment process differs from the partial adjustment process (Fisher, 1961) in that the speed of adjustment is not constant but changes according to the product's lifecycle. The proposed adjustment process takes the form of a logistic equation (Escobido, 2010) to allow us to model the evolution of the competition throughout the product lifecycle.

The above extensions enable us to derive coupled equations governing the dynamics of competition between the differentiated products. The rest of the paper is organized to deduce the dynamics of these equations and the implications on the competition. The succeeding section (Research Model) lays down the assumptions and builds the model. In Method and Findings, we use the tools on nonlinear systems to characterize the dynamics of the competition. Specifically, we obtain the equilibrium points and find the conditions of stability of these points. In the Discussion section, we develop the quality and substitutability diagram to visually capture important aspects of the dynamics of competition. Lastly we conclude this paper with a brief discussion of the Managerial Implications of our results, highlighting the relevance in helping managers understand the implications of the different aspects of differentiation in the competition between products.

Research Model

Building upon previous work by Dixit (Dixit, 1979) and Singh and Vives (Singh & Vives, 1984) , we consider a duopoly model where firms produce differentiated products. The representative consumer's utility is a function of the consumption of the two differentiated products and the numeraire product I as given by

$$U(q_1, q_2, I) = \alpha_1 q_1 + \alpha_2 q_2 - \frac{1}{2}(q_1^2 + 2\gamma q_1 q_2 + q_2^2) + I \tag{1}$$

where q_i $(i = 1,2)$ denote the consumer's consumption of the product i. Two aspects of differentiation are captured in the parameters α and γ. The parameter $\alpha_i > 0$ measures quality in the vertical sense in that an increase in α_i increases the marginal utility of consuming product i. The γ measures the cross-price effects owing to the substitutability between the products and can either be positive, negative or zero depending on whether the products are substitutes ($\gamma > 0$), complements ($\gamma < 0$) or independent ($\gamma = 0$). The focus of this article is when the products are substitutes.

The representative consumer maximize $U(q_1, q_2, I) - \sum_{i=1}^{2} p_i q_i$ where q_i is the amount of product i and p_i its price to yield the inverse demand

$$\begin{aligned} p_1 &= \alpha_1 - q_1 - \gamma q_2 \\ p_2 &= \alpha_2 - q_2 - \gamma q_1 \end{aligned} \tag{2}$$

The direct demand can then be obtained as:

$$\begin{aligned} q_1 &= a_1 - b_1 p_1 + d p_2 \\ q_2 &= a_2 - b_2 p_2 + d p_1 \end{aligned} \tag{3}$$

where $\alpha_i = \frac{\alpha_i - \alpha_j\gamma}{\delta}$, $b_i = \frac{1}{\delta}$ for $i \neq j, i = 1{,}2$ and $d = \frac{\gamma}{\delta}, \delta = 1 - \gamma^2$. The concavity of the utility function requires $\gamma \leq 1$ and we restrict ourselves to quantities where prices are positive.

For firms producing the products under Cournot competition, they adjust their quantities to maximize their profits knowing the implications on the price as given by (2). The revenue in selling q_i at price p_i is:

$$revenue_i(q_i) = p_i q_i \tag{4}$$

Assuming a fixed cost f_i and a constant marginal cost m_i, the cost is:

$$cost_i(q_i) = f_i + m_i q_i \tag{5}$$

The profit $\pi_i(q_i)$ for the firm i given the output of the other firm j then is:

$$\pi_i(q_i) = (\alpha_i - m_i - \gamma q_j)q_i - f_i \tag{6}$$

From here on, we will consider profits net of marginal cost by setting $m_i = 0$. We can always revert to cases with constant marginal cost by replacing α_i with $\alpha_i - m_i$. Maximizing this profit, assuming that the other firms keeps their quantity constant (Cournot conjecture, $\frac{\partial q_j}{\partial q_i} = 0$), gives the reaction function or best response function of firm i:

$$q_i^*(q_j) = \frac{\alpha_i - \gamma q_j}{2} \tag{7}$$

The reaction function of the two firms intersect to yield a Cournot-Nash equilibrium where each firm will not have an incentive to change its output, given the output of the other.

So far the above discussion is a static case of decision making for a firm given the output of the other firms. For the firm i to update its initial output q_{i0} to the desired level $q_i^*(q_j)$ given in (7), it needs to have an adjustment process. Repeating this adjustment process results in a dynamic scenario where the firms continuously update their output levels to maximize their profits.

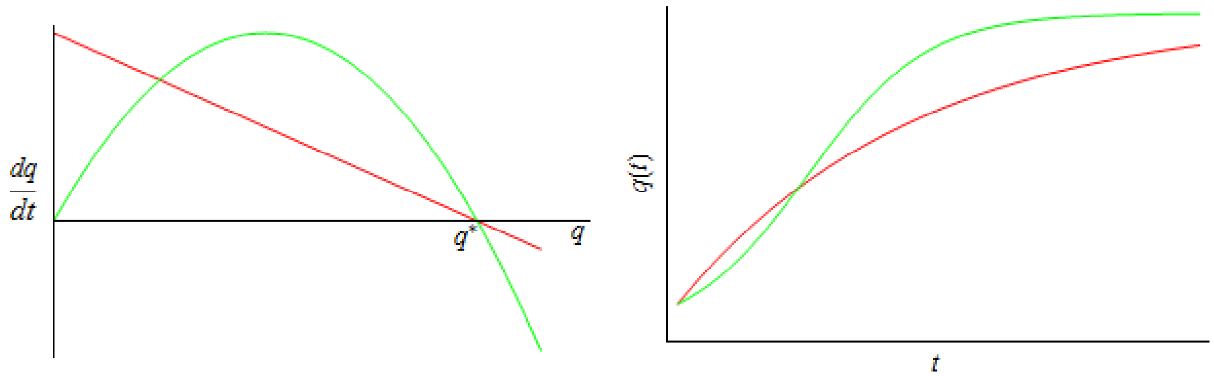
The naïve choice of adjustment process is where firms adjust their output level to the discrepancy of the desired output $q_i^*(q_j)$ and the actual level $q_i(t)$ at that instant (Theocharis, 1960). In this model, the firms ramp to the desired level irrespective of the discrepancy. This ability may be difficult especially in the early stages of the industry where capacity may be constrained or demand may not be realized. A refinement to this is the partial adjustment process where firms continuously adjust their output level in proportion to the discrepancy. This is given by $\frac{dq_i(t)}{dt} = k_i(q_i^*(q_j) - q_i(t))$ where $k_i > 0$ is called the speed of adjustment of firm i (Fisher, 1961). The speed of adjustment is constant and $k = 1$ reverts to the naïve case.

Analysis of the dynamics using partial adjustment process result in firms coexisting and converging to their desired output levels given in (7) independent of the speed of adjustment (Fisher, 1961). Empirical data over the industry life cycle, however, show a number of firms fail, unable to coexist with the other firms. Competition intensifies and the number of firms exiting the industry increases before eventually settling with a few dominant firms (see for instance (Gort & Klepper, 1982)).

Instead of the partial adjustment process with constant speed of adjustment, we imbue the firms' knowledge of the product lifecycle. This means that firms update their output following the product's life cycle where the growth rate of the output varies depending on the phase the product is in (Escobido, 2010). Early in the lifecycle, the output is small as the product is still in the introductory phase. As the product moves to a growth phase, the output increases and the adjustment process rises with increasing output up to a maximum. The adjustment then falls to zero at the desired equilibrium level and becoming increasingly negative when the output exceeds the desired level. Compared to the partial adjustment process, the rate of change of the output is not constant but changes as the quantities change. This behavior is illustrated in Figure 1a. This adjustment behavior can be modeled by a logistic equation:

$$\frac{dq_i(t)}{dt} = k_i(q_i^*(q_j) - q_i(t))q_i(t) \tag{8}$$

If the desired level is constant over time, the output would follow the familiar S-curve with the desired level as its asymptotic limit as shown Figure 1b:



(a) (b)

Figure 1. (a) Plot of dq/dt vs. q for the partial (red) and logistic adjustment processes (green). (b) Time evolution of $q(t)$ for both processes

If the firms adopt the logistic adjustment process, from (6) and (7), the resulting dynamical equations become:

$$\frac{dq_i(t)}{dt} = k_i\left(\frac{\alpha_i - \gamma q_j}{2} - q_i\right)q_i \tag{9}$$

Or explicitly:

$$\begin{aligned} \frac{dq_1(t)}{dt} &= k_1 q_1\left(\frac{\alpha_1}{2} - q_1 - \frac{\gamma}{2}q_2\right) \\ \frac{dq_2(t)}{dt} &= k_2 q_2\left(\frac{\alpha_2}{2} - q_2 - \frac{\gamma}{2}q_1\right) \end{aligned} \tag{10}$$

Equation (10) is formally equivalent to Lotka-Volterra competition equations popular in population ecology (see for instance (Murray, 1989)).

Methods and Findings

The dynamics of the coupled equations given by (10) can be understood by looking at the terms. The third term in (10) involves the multiplication of q_1 and q_2 and denotes the interaction between the products. If the products are perfectly differentiated ($\gamma = 0$) then the contribution of these terms are zero. This means that the products do not interact and the evolution of the output follows the logistic pattern where the output initially grows exponentially at the rate $k_i\alpha_i/2$. The rate of growth depends greatly on the product quality α_i -the higher it is, the faster is the growth.

As the output grows, however, the second term q_i^2 becomes larger and decreases the growth rate. This happens because the maximum desired level it is going to produce is its monopoly output $\frac{\alpha_i}{2}$. Assuming that the quality α_i does not change in time, the monopoly output remains the same. As such as q_i's increase, the discrepancy to the desired level ($q_i^* - q_i$) for it to maximize its profit becomes less.

If the products are substitutes ($\gamma > 0$), then the interaction term $\frac{\gamma}{2}q_1q_2$ measures the detrimental effect of the competition owing to the substitutability of the other product. The effect of the competition by other products would be to reduce the growth rate of the production of q_i's. The more substitutable the products are (higher γ), the stiffer the competition becomes (higher $\gamma/2$) and the greater the reduction in the growth rate may become.

Given the competition setting above, the dynamics can be understood using concepts from nonlinear systems. Prominent concepts in the analysis would be the equilibrium points and stability conditions. The long-run equilibrium outcomes (q_1^*, q_2^*) can be obtained from $\frac{dq_i(t)}{dt} = 0$. From (10), we obtain four equilibrium points relating to both products having no outputs $(0,0)$, one product gaining monopoly and the other exiting the market (i.e. $(\alpha_1/2, 0)$ or $(0, \alpha_2/2)$) or both products coexisting $\left(\frac{2\alpha_1 - \gamma\alpha_2}{4-\gamma^2}, \frac{2\alpha_2 - \gamma\alpha_1}{4-\gamma^2}\right)$.

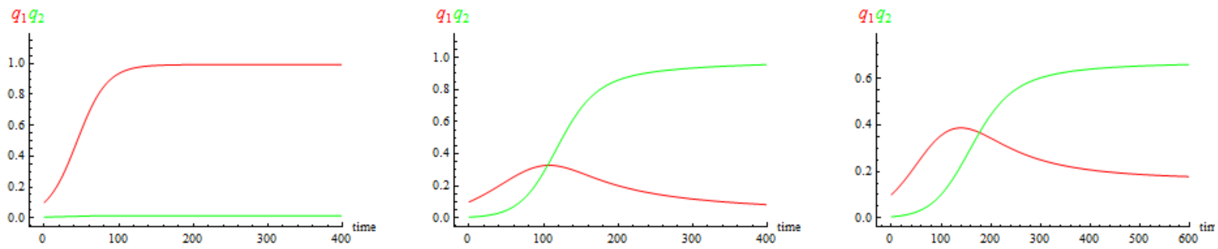
For these equilibrium points to persist, we need to check their stability. They are stable if the eigenvalues of the Jacobian evaluated at these points have negative real parts. These eigenvalues can be obtained by polynomial expansion of the characteristic equation. The necessary and sufficient condition for a polynomial to have all roots with negative real parts is given by the Routh-Hurwitz criterion (Gradshteyn & Ryzhik, 2007). The resulting equilibrium points and their stability are tabulated below:

Table 1. Equilibrium points and their stability

Equilibrium Points (q_1^*, q_2^*)	**Stable if**
(0,0)	Never
$(0, \frac{\alpha_2}{2})$	$\alpha_1 < \frac{\gamma}{2}\alpha_2$
$(\frac{\alpha_1}{2}, 0)$	$\alpha_2 < \frac{\gamma}{2}\alpha_1$
$\left(\frac{2\alpha_1 - \gamma\alpha_2}{4-\gamma^2}, \frac{2\alpha_2 - \gamma\alpha_1}{4-\gamma^2}\right)$	$\alpha_1 > \frac{\gamma}{2}\alpha_2$ and $\alpha_2 > \frac{\gamma}{2}\alpha_1$

Intuitively, the condition for the stability of the coexistence of the products can be understood by considering (10) with $q_1 \approx 0$. When q_1 is small, q_2 approaches its equilibrium level q_2^* which is just its monopoly output $\frac{\alpha_2}{2}$. q_1 will grow provided $\frac{\alpha_1}{2} > \frac{\gamma}{2} q_2^* = \frac{\gamma}{2}\frac{\alpha_2}{2}$. Similarly, q_2 when small, will grow provided $\frac{\alpha_2}{2} > \frac{\gamma}{2}\frac{\alpha_1}{2}$. As such, both firms can coexist if $\alpha_1 > \frac{\gamma}{2}\alpha_2$ and $\alpha_2 > \frac{\gamma}{2}\alpha_1$.

The evolution of the competition would vary depending on which condition it satisfies. Though there are only four long-run outcomes, there are infinite paths to these outcomes depending on the actual parameters and initial conditions. Examples of the evolution of competition for the different equilibrium points (excluding the trivial (*0,0*) point) are illustrated in the figure below:



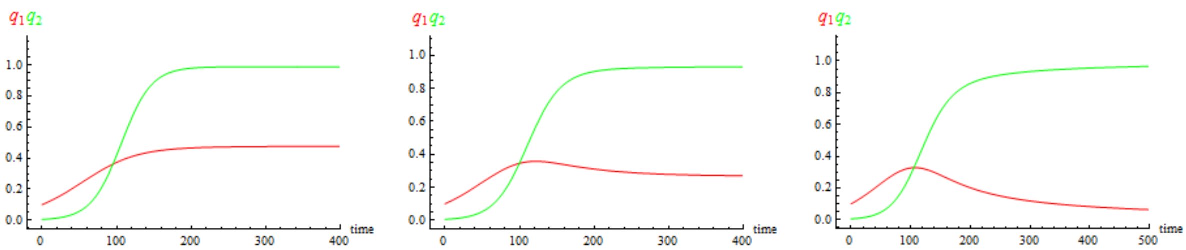
(a) Product 1 wins (b) Product 2 wins (c) Both Products Coexist

Figure 2. Evolution of competition (*α changing*). (a)$\boldsymbol{\alpha_1 = 2, \alpha_2 = 1}$ (b) $\boldsymbol{\alpha_1 = 1, \alpha_2 = 2}$ (c) $\boldsymbol{\alpha_1 = 1.5, \alpha_2 = 2}$. Initial condition for all cases: $\gamma = 1, k_1 = k_2 = 0.05, q_{1_o} = 0.1, q_{2_o} = 0.005$

By having a better quality and initial output, product 1 in (a) is quick to monopolize the market. In (b), albeit of smaller initial output, product 2's better quality eventually wins the market in the long run. (c) Though they have different quality, the difference is not sufficient to monopolize the market and both products coexist.

Figure 3 below considers the case where the substitutability between the products changes. Even though they differ in quality, the substitutability in (a) is very low to permit the coexistence of both products. Because they are almost completely differentiated, both products grow to their monopoly levels.

As the substitutability of the products increase, the one with lower quality loses more market share but still survives in the competition as shown in (b). In (c), the products become completely substitutable and product 2 wins the head-on competition with its superior quality.



(a) Both products coexist (b) Both products coexist (

c) Product 2 wins

Figure 3. Evolution of competition (γ *changing*). (a) $\gamma = 0.05$ (b) $\gamma = 0.5$ (c) $\gamma = 1$. Initial condition for all cases: $\alpha_1 = 1, \alpha_2 = 2, k_1 = k_2 = 0.05, q_{1_o} = 0.1, q_{2_o} = 0.005$

Discussion

The conditions for coexistence $\alpha_1 > \frac{\gamma}{2}\alpha_2$ and $\alpha_2 > \frac{\gamma}{2}\alpha_1$ define a two-dimensional cone within which products coexist. Outside of this cone, the conditions $\alpha_1 < \frac{\gamma}{2}\alpha_2$ and $\alpha_2 < \frac{\gamma}{2}\alpha_1$ hold which means the success of one and the failure of the other product. The figure below illustrates this:

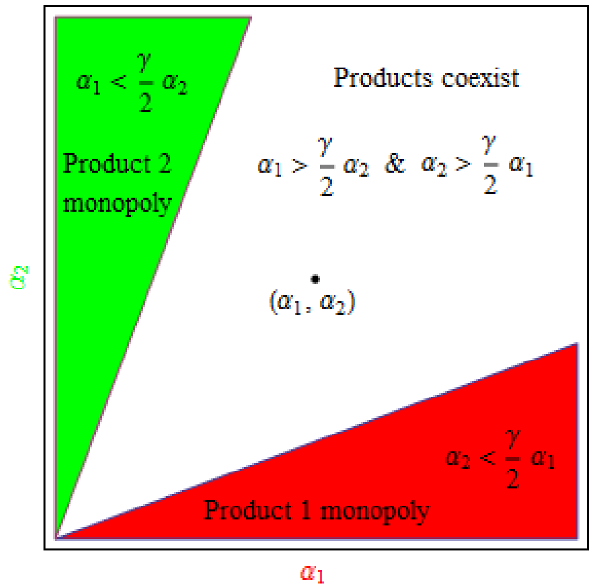
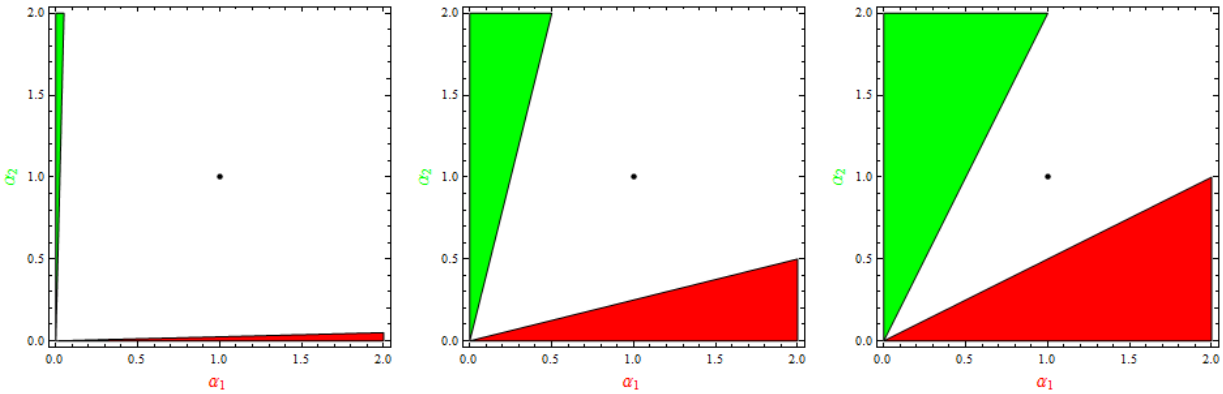


Figure 4. Products vertical and horizontal differentiation and competition regions

The angle of the cone of coexistence where products can coexist is given by

$$\theta = \cos^{-1}\left(\frac{4\gamma}{4+\gamma^2}\right) \tag{11}$$

This means that if products are independent ($\gamma = 0$), both products coexist ($\theta = 90^\circ$) and the quantities grow up to their monopoly output. As products become more substitutable ($\gamma > 0$), the angle decreases, contracting the region of coexistence. This behavior is illustrated in a series of figures below:



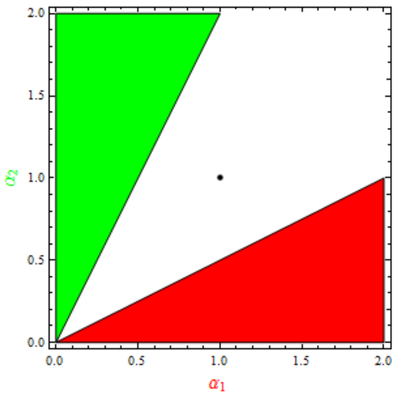
(a) $\gamma = 0.05$ (b) $\gamma = 0.5$ (c) $\gamma = 1$

Figure 5. Impact of horizontal differentiation (substitutability) on competition. Parameter settings $\boldsymbol{\alpha_1 = \alpha_2 = 1, k_1 = k_2 = 0.05}$.

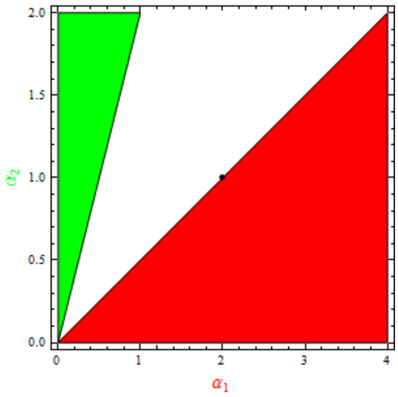
Figure 5 (a) shows that coexistence region is large for highly differentiated products (low γ). In (b) as the differentiation decreases (higher γ), the region of coexistence contracts. When products are not differentiated ($\gamma = 1$) as in (c), the coexistence region is the smallest.

It is interesting to note that as the products become completely substitutable, the coexistence region does not collapse along the diagonal. Concavity of the utility function limits the $\gamma \leq 1$. When the products become completely substitutable $\gamma = 1$, the minimum angle for the coexistence region would be $\cos^{-1}\left(\frac{4}{5}\right)$ which is about 37°. The rate at which a firm i is reducing output level per output $\frac{1}{q_i}\frac{dq_i}{dt}$ because of competition is $\frac{\gamma}{2}q_j$ and if the products are identical ($\alpha_1 = \alpha_2$ *and* $\gamma = 1$), this would just be half of its competitor's output. As such, substitutability by itself may not necessarily win the competition especially if the competition is appropriately situated.

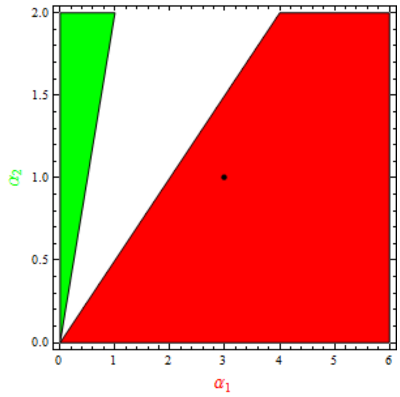
What then can knock-off competition? An asymmetric increase in quality is needed to win over the competition. An extreme case where the other product (in this case product 2) is not improving on its quality is illustrated in the figure below:



(a) $\alpha_1 = \alpha_2 = 1$



(b) $\alpha_1 = 2, \alpha_2 = 1$



(c) $\alpha_1 = 3, \alpha_2 = 1$

Figure 6. Impact of vertical differentiation (quality) on competition.

Even though the products are substitutable to start with in (a), an increase in the quality of product 1 in (b) makes the customer prefer it more, so much so that it tipped the market in its favor. Additional increase in (c) will further its hold on the market and provides a barrier that the other firm needs to overcome if its wants to remain in the market.

A more likely scenario though would be both firms improving on the quality of its product. In such case, what matters more is the difference between the quality and not how much it has improved. For even if it has improved a lot, but relative to the competition their difference is not large enough, both firms can just coexist as illustrated in Figure 2c above.

Managerial Implications

The foregoing discussion clearly delineates the effects of the two aspects of differentiation covered in this article. Horizontal differentiation (substitutability) reduces the directness of competition by providing a larger cone of coexistence between the products. The less substitutable the products are, the larger this cone of coexistence. This effect though is symmetric in the sense that a similar breathing space is given to the competing product.

The effect of differences in vertical differentiation (quality) is more dramatic. In the short term, the growth rate is driven by the quality of the product. Even though the product is differentiated in the low-substitutability sense, consumers would prefer it the higher its quality.

In the long term, higher quality enables a product to encroach into the market of the competition when both are substitutes.

Deciding which to prioritize, quality or substitutability would depend on the objective of the product in the market. If it would be as a niche product coexisting with other products in the market, horizontal differentiation (less substitutable) would play a key role. If the intent however is to gain more market share by encroaching on the market of competing products, less differentiation (more substitutable) and more quality will do the trick.

These decisions though are not one-off as firms continuously improve their offerings and making the competition dynamic. The quality – substitutability diagram can aid the managers by plotting the trajectory of quality improvements and adjusting the competition regions accordingly. From these, one can read off possible implications of the differentiation aspects and the likely outcome of the competition.

References

Bain, J. S. (1956). *Barriers to new competition; their character and consequences in manufacturing industries.* Cambridge, MA: Harvard University Press.

Beath, J., & Katsoulacos, Y. (1991). *The economic theory of product differentiation.* Cambridge: Cambridge University Press.

Dixit, A. (1979). A Model of Duopoly Suggesting a Theory of Entry Barriers. *Bell Journal of Economics , 10*, 20-32.

Escobido, M. (2010). Duopoly dynamics under logistic adjustment. *In Preparation*.

Fisher, F. M. (1961). The Stability of the cournot Oligopoly solution: The Effects of Speeds of Adjustment and Increasing Marginal Costs. *The Review of Economic Studies , 28* (2), 125-135.

Gort, M., & Klepper, S. (1982). Time Paths in the Diffusion of Product Innovations. *The Economic Journal , 92* (367), 630-653.

Gradshteyn, I. S., & Ryzhik, I. M. (2007). *Tables of Integrals, Series, and Products* (7th ed.). (A. Jeffrey, & D. Zwillinger, Eds.) London: Elsevier.

Kahn, K. B. (Ed.). (2005). *The PDMA Handbook of New Product Development* (Second ed.). Hoboken, NJ: John Wiley & Sons.

Murray, J. (1989). *Mathematical Biology.* Berlin: Springer.

Singh, N., & Vives, X. (1984). Price and Quantity Competition in a Differentiated Duopoly. *The RAND Journal of Economics , 15*, 546-554.

Theocharis, R. (1960). On the Stability of the Cournot Solution on the Oligopoly Problem. *The Review of Economic Studies , 27* (2), 133-134.

Tirole, J. (1988). *The theory of industrial organization.* Cambridge, MA: MIT Press.